

Image Enhancement in the Spatial Domain

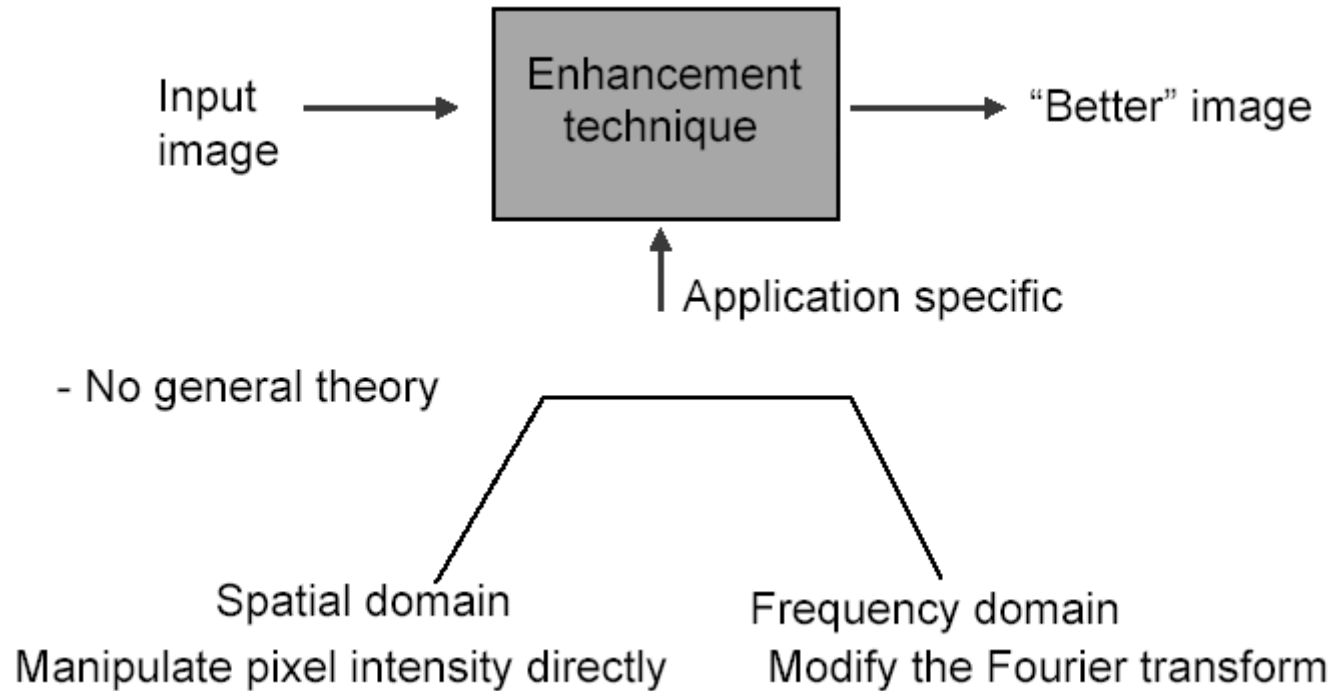
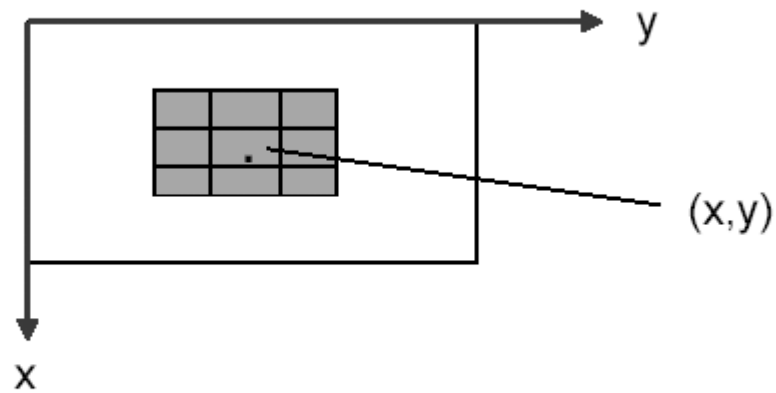


Image Enhancement in the Spatial Domain

$$g(x,y) = T[f(x,y)]$$

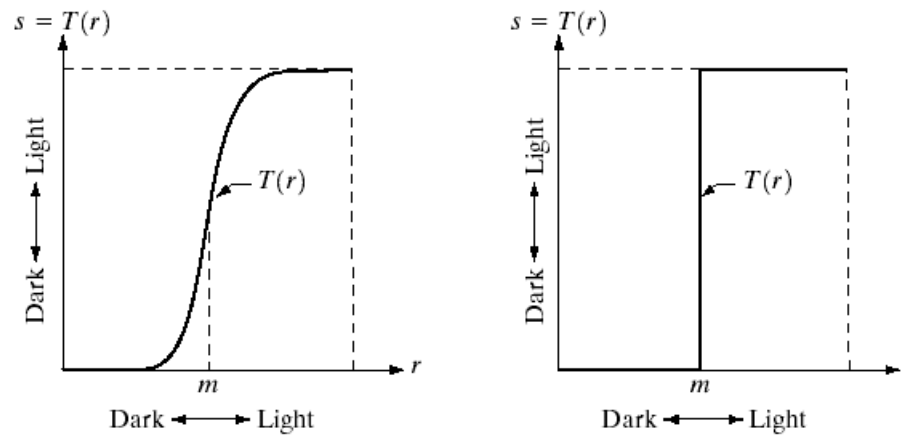


Simplest case: Neighbourhood is (x,y)

[$g(.)$ depends only on the value of f at (x,y)]

Image Enhancement in the Spatial Domain

Gray Level Transformation Functions



a b

FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Image Enhancement in the Spatial Domain

Gray Level Transformation Functions

FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

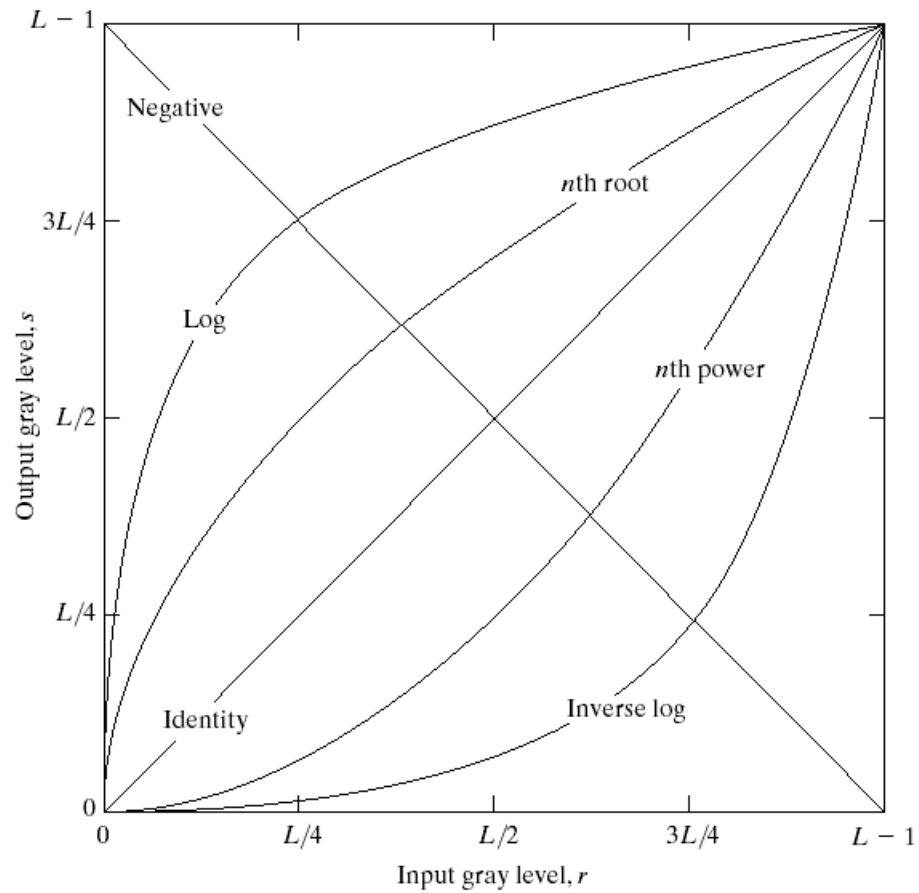


Image Enhancement in the Spatial Domain

(a) Negative image: Example: $g(x,y) = 255 - f(x,y)$

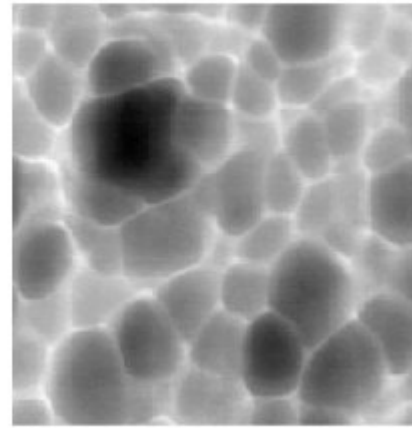
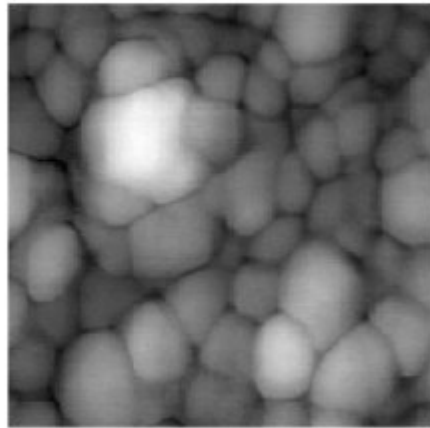
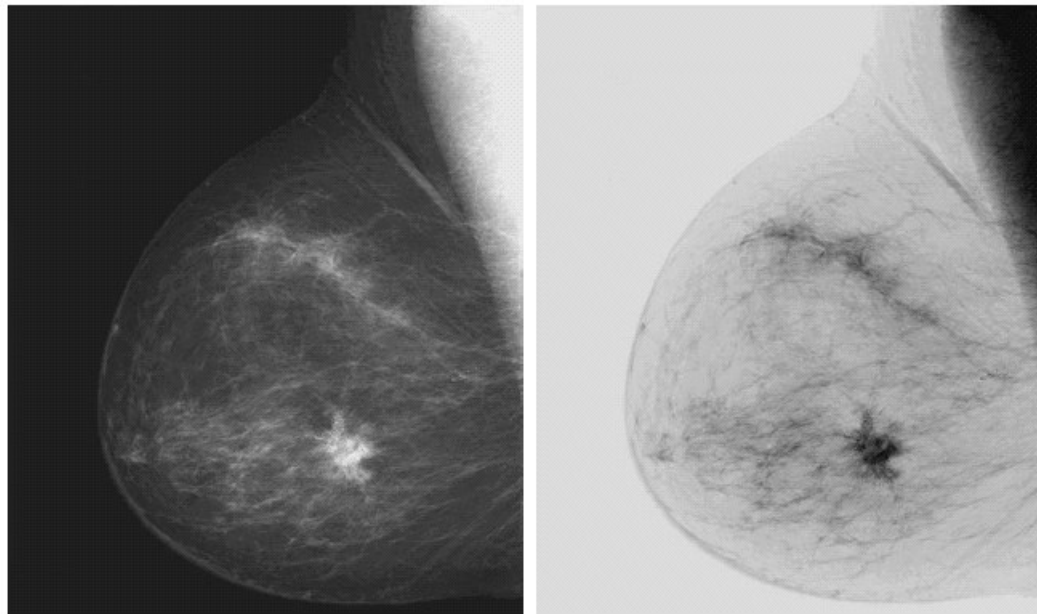


Image Enhancement in the Spatial Domain

Negative Image: Another Example



a b

FIGURE 3.4

(a) Original digital mammogram.
(b) Negative image obtained using the negative transformation in Eq. (3.2-1).
(Courtesy of G.E. Medical Systems.)

Image Enhancement in the Spatial Domain

(c) Compressing dynamic range

$$s = c \log (1 + |r|) \quad c \longrightarrow \text{Scaling factor}$$

Example: Displaying the Fourier Spectrum

a b

FIGURE 3.5

(a) Fourier spectrum.
(b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

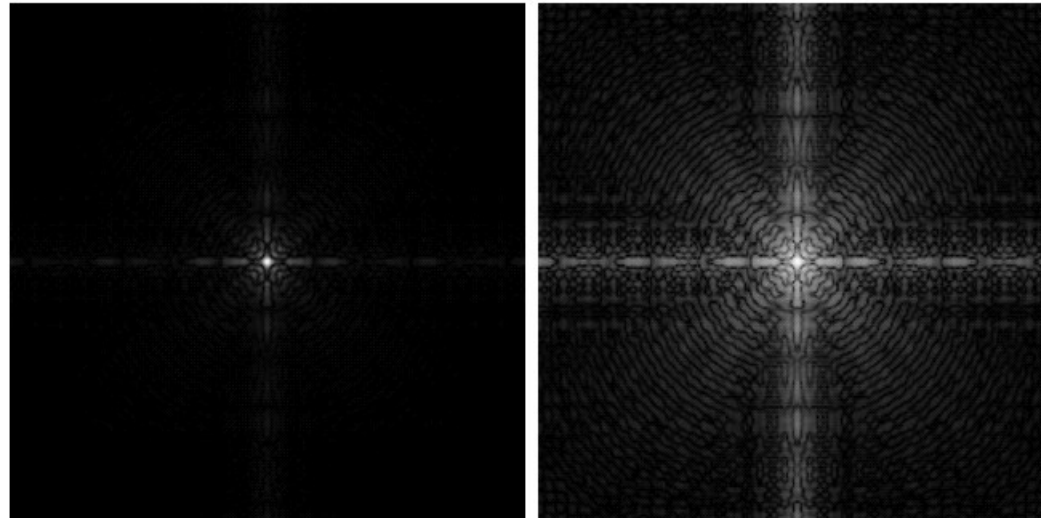


Image Enhancement in the Spatial Domain

Power Function

$$S = cR^\gamma$$

C and γ are positive constants.

Often referred to as “gamma correction”.

CRT –intensity-to-voltage response follows a power function (typical value of gamma in the range 1.5-2.5.)

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Power Function

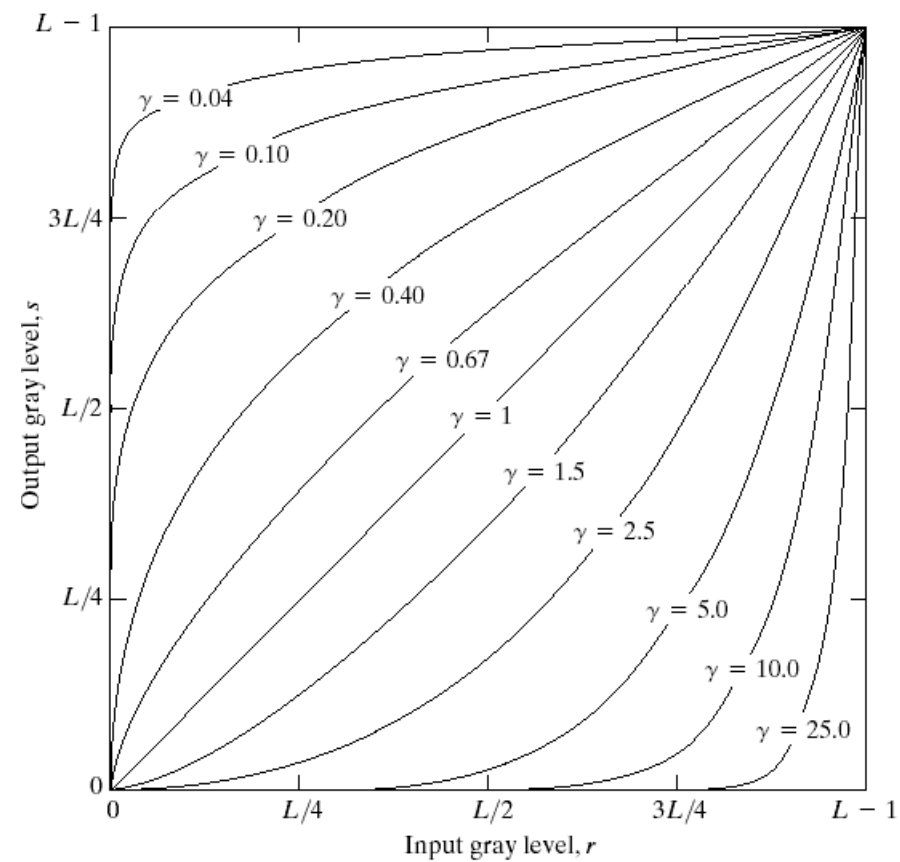


FIGURE 3.6 Plots of the equation $s = cr^\gamma$ for various values of γ ($c = 1$ in all cases).

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Power Function: Example

a b
c d

FIGURE 3.9
(a) Aerial image.
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and $5.0,$ respectively. (Original image for this example courtesy of NASA.)



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Power Function: Gamma Correction

a b
c d

FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

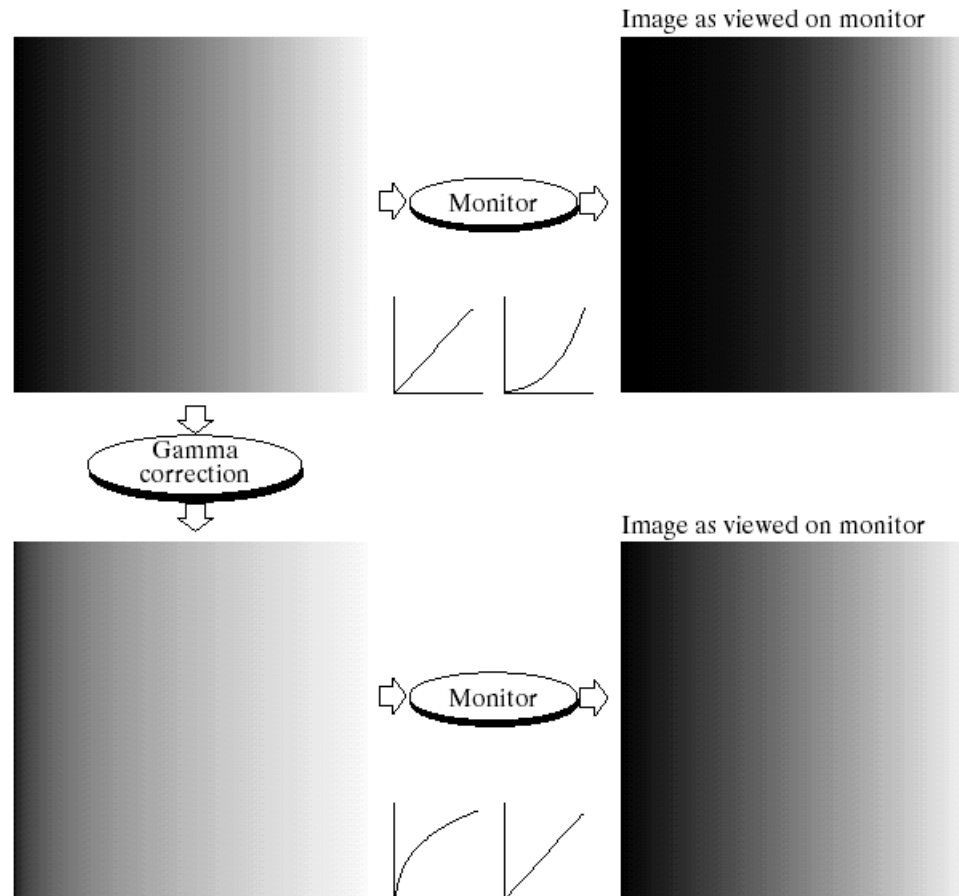
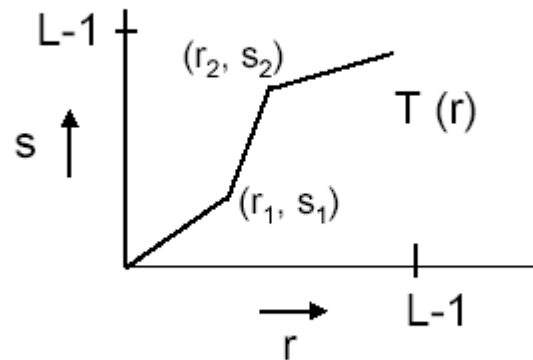


Image Enhancement in the Spatial Domain

(b) Contrast stretching



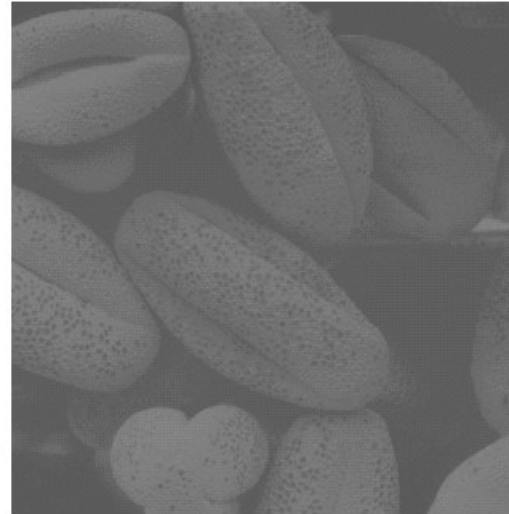
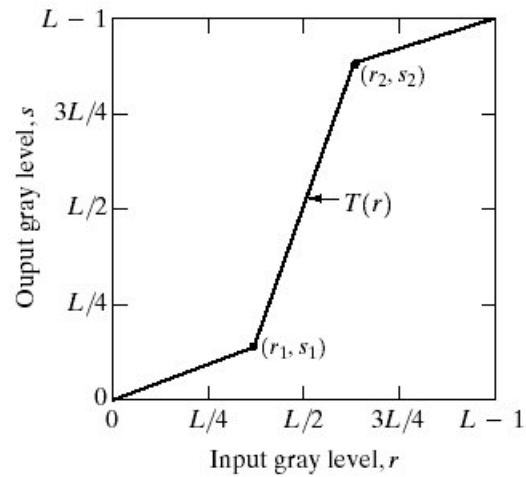
$$\begin{matrix} r_1 = s_1 \\ r_2 = s_2 \end{matrix}$$

no change

$$\begin{matrix} r_1 = r_2 \\ s_1 = 0 \\ s_2 = L-1 \end{matrix}$$

Thresholding
at r_1

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a b
c d

FIGURE 3.10

Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

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Histogram

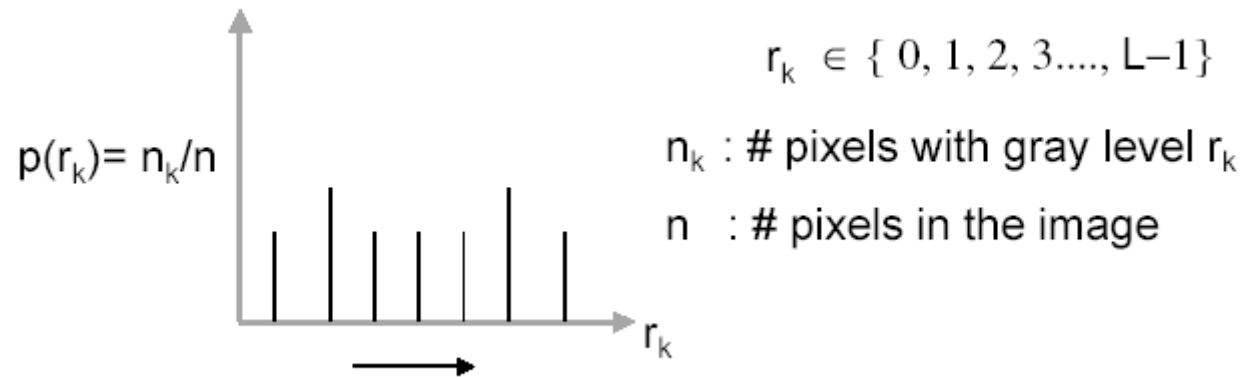


Image Enhancement in the Spatial Domain

Histogram: Examples

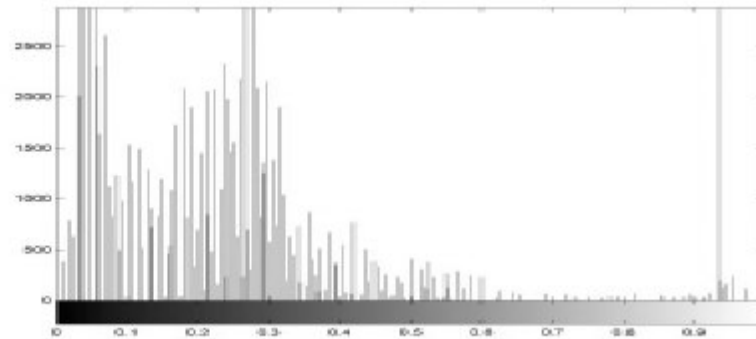
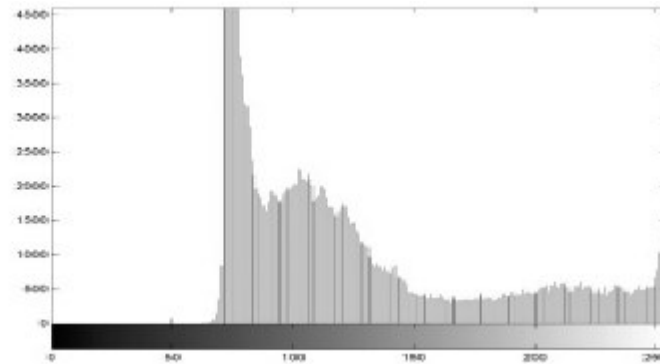
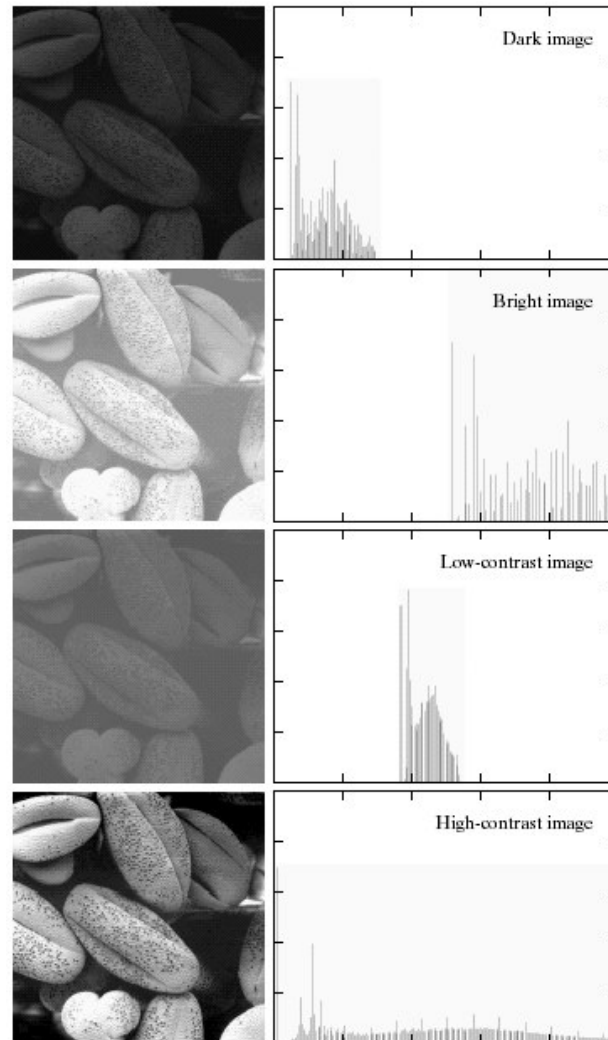


Image Enhancement in the Spatial Domain

Histogram: Examples



a b

FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Image Enhancement in the Spatial Domain

Histogram Equalization

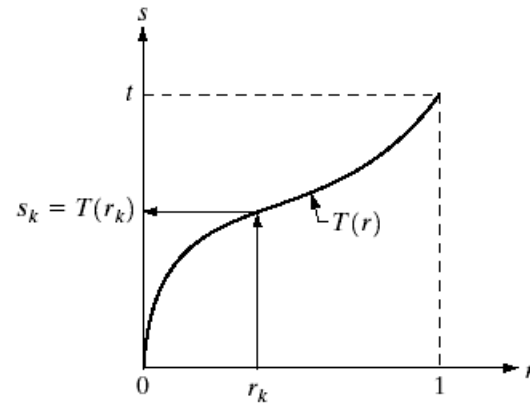


FIGURE 3.16 A gray-level transformation function that is both single valued and monotonically increasing.

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Histogram Equalization

(i) $T(r)$ is single valued and monotonically increasing in
 $0 \leq r \leq 1$

(ii) $0 \leq T(r) \leq 1$ for $0 \leq r \leq 1$

$$[0, 1] \xrightarrow{T} [0, 1]$$

Inverse transformation : $T^{-1}(s) = r$ $0 \leq s \leq 1$

$T^{-1}(s)$ also satisfies (i) and (ii)

The gray levels in the image can be viewed as random variables taking values in the range $[0,1]$.

Let $p_r(r)$: p.d.f. of input level r and let $p_s(s)$: p.d.f. of s

Image Enhancement in the Spatial Domain

Histogram Equalization

r : Input gray level $\in [0, 1]$

s : Transformed gray level $\in [0, 1]$

$$s = T(r) \quad T : \text{Transformation function}$$

We are interested in obtaining a transformation function $T(\cdot)$ which transforms an arbitrary p.d.f. to an uniform distribution

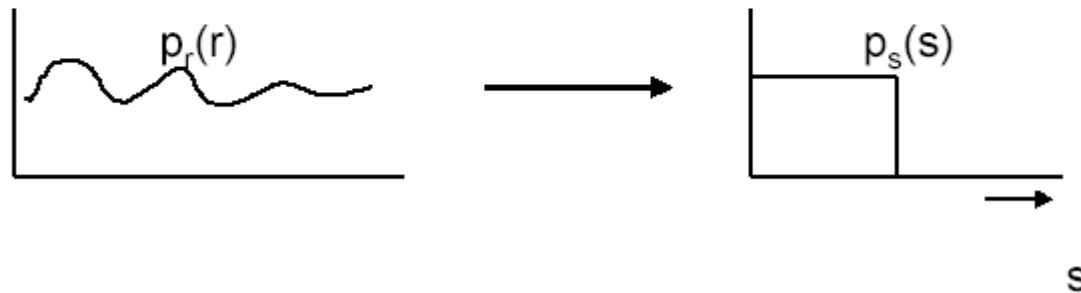


Image Enhancement in the Spatial Domain

Histogram Equalization

$$\text{Consider } s = T(r) = \int_0^r p_r(w) dw \quad 0 \leq r \leq 1$$

(Cumulative distribution function of r)

$$p_s(s) = p_r(r) \left. \frac{dr}{ds} \right|_{r=T^{-1}(s)} ;$$

$$\frac{ds}{dr} = \frac{d}{dr} \left[\int_r^0 p_r(w) dw \right] = p_r(r)$$

$$\therefore p_s(s) = p_r(r) \left. \frac{1}{p_r(r)} \right|_{r=T^{-1}(s)} \equiv 1 \quad 0 \leq s \leq 1$$

Image Enhancement in the Spatial Domain

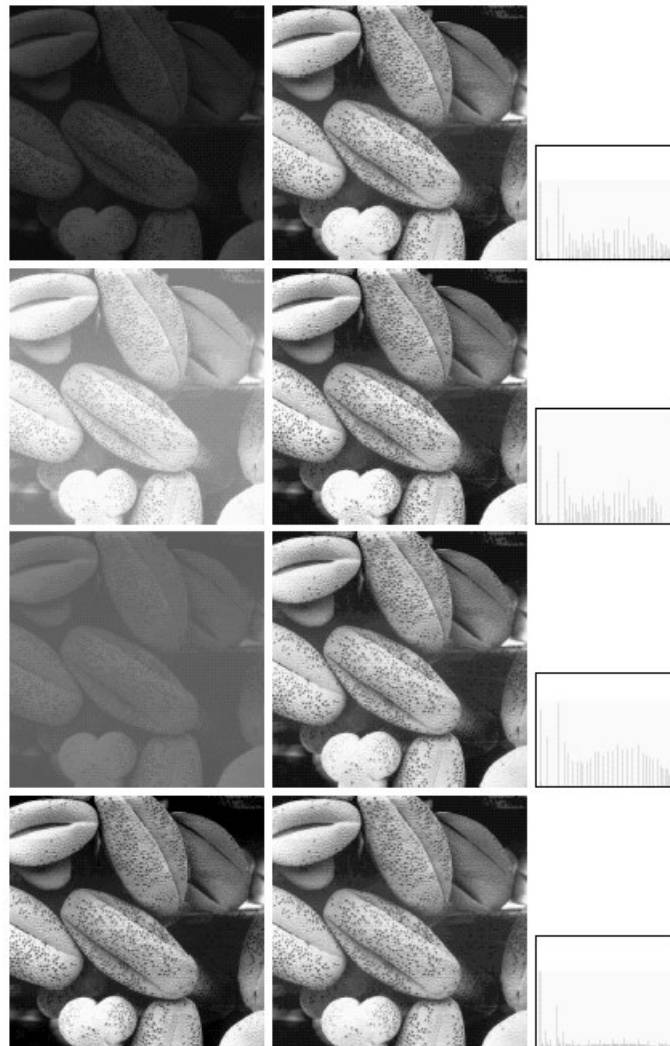
Histogram Equalization

$$p_r(r_k) = \frac{n_k}{n} \quad 0 \leq r_k \leq 1 \quad ; \quad k = 0, 1, \dots, L-1$$

$L \rightarrow$ Number of levels

$$s_k = T(r_k) = \sum_{j=0}^k p_r(r_j) = \sum_{j=0}^k \frac{n_j}{n}$$

Image Enhancement in the Spatial Domain



Histogram Equalization
: Examples

a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

Image Enhancement in the Spatial Domain

Histogram Equalization: Examples

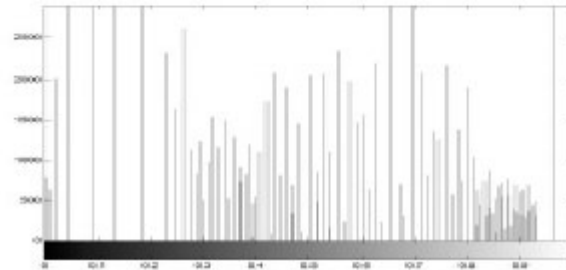
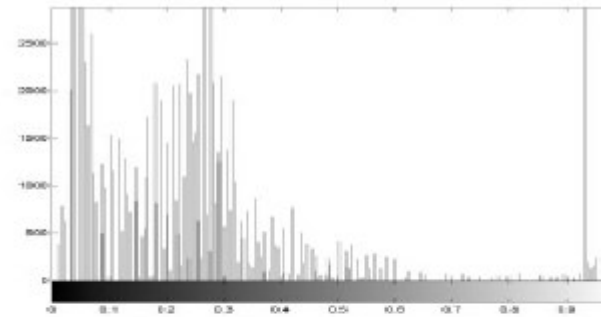


Image Enhancement in the Spatial Domain

Histogram Specification

$$\text{Suppose } s = T(r) = \int_0^r p_r(w) dw$$

$p_r(r) \rightarrow$ Original histogram ; $p_z(z) \rightarrow$ Desired histogram

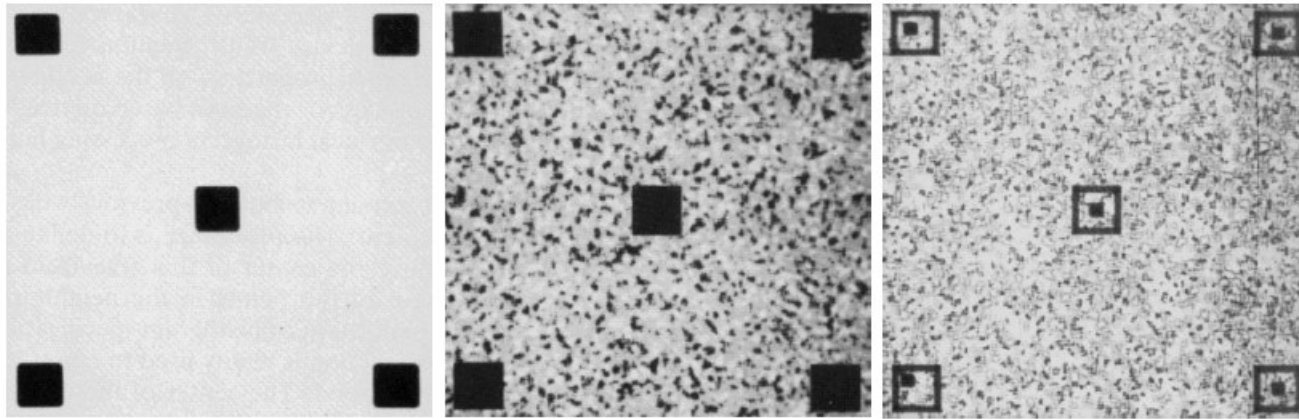
$$\text{Let } v = G(z) = \int_0^z p_z(w) dw \quad \text{and} \quad z = G^{-1}(v)$$

But s and v are identical p.d.f.

$$\therefore z = G^{-1}(v) = G^{-1}(s) = G^{-1}(T(r))$$

Image Enhancement in the Spatial Domain

Histogram Equalization: Local Enhancement



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

Image Enhancement in the Spatial Domain

Spatial Filtering

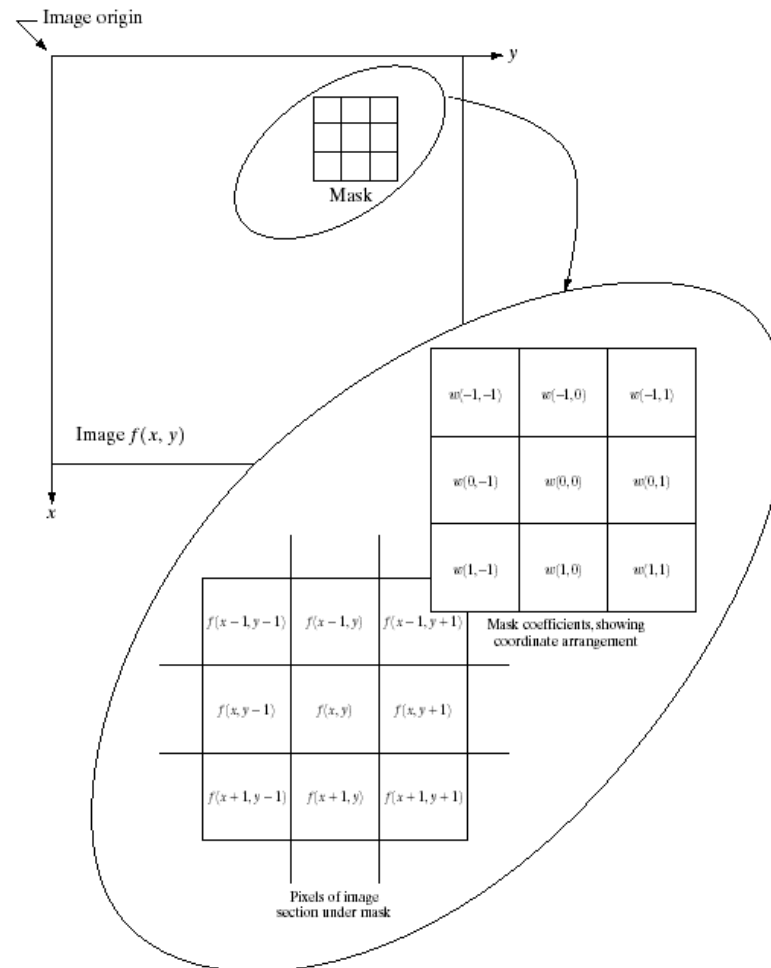
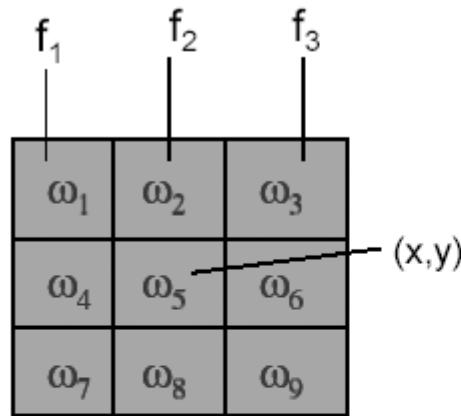


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

Image Enhancement in the Spatial Domain

Spatial Filtering



Replace $f(x,y)$ with

$$\hat{f}(x,y) = \sum_i \omega_i f_i$$

Linear filter

LPF: reduces additive noise → blurs the image
→ sharpness details are lost
(Example: Local averaging)

Image Enhancement in the Spatial Domain

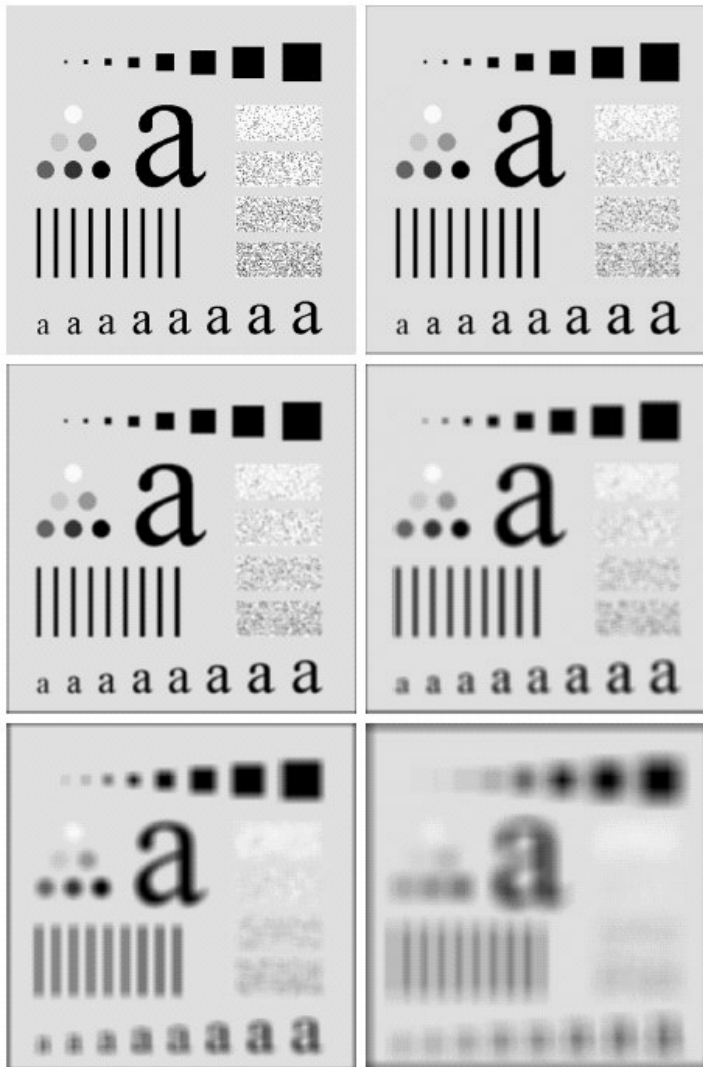
Spatial Filtering: Neighborhood Averaging

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

Image Enhancement in the Spatial Domain



Spatial Filtering:
Neighborhood Averaging

FIGURE 3.35 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15,$ and 35 , respectively. The black squares at the top are of sizes $n = 3, 5, 9, 15, 25, 35, 45,$ and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

Image Enhancement in the Spatial Domain

Median Filter

Replace $f(x,y)$ with $\text{median} [f(x', y')]$
 $(x', y') \in \mathcal{E}$ neighbourhood

- Useful in eliminating intensity spikes. (salt & pepper noise)
- Better at preserving edges.

Example:

10	20	20
20	15	20
25	20	100

→ (10,15,20,20,20,20,20,25,100)

Median=20

So replace (15) with (20)

Image Enhancement in the Spatial Domain

Median Filter: Example



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Sharpening Filter

- Enhance finer image details (such as edges)
- Detect region /object boundaries.

Example:

-1	-1	-1
-1	8	-1
-1	-1	-1

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Highboost Filter

0	-1	0	-1	-1	-1
-1	$A + 4$	-1	-1	$A + 8$	-1
0	-1	0	-1	-1	-1

a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

Image Enhancement in the Spatial Domain

Highboost Filter: Example

a b
c d

FIGURE 3.43

(a) Same as Fig. 3.41(c), but darker.

(a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.

(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

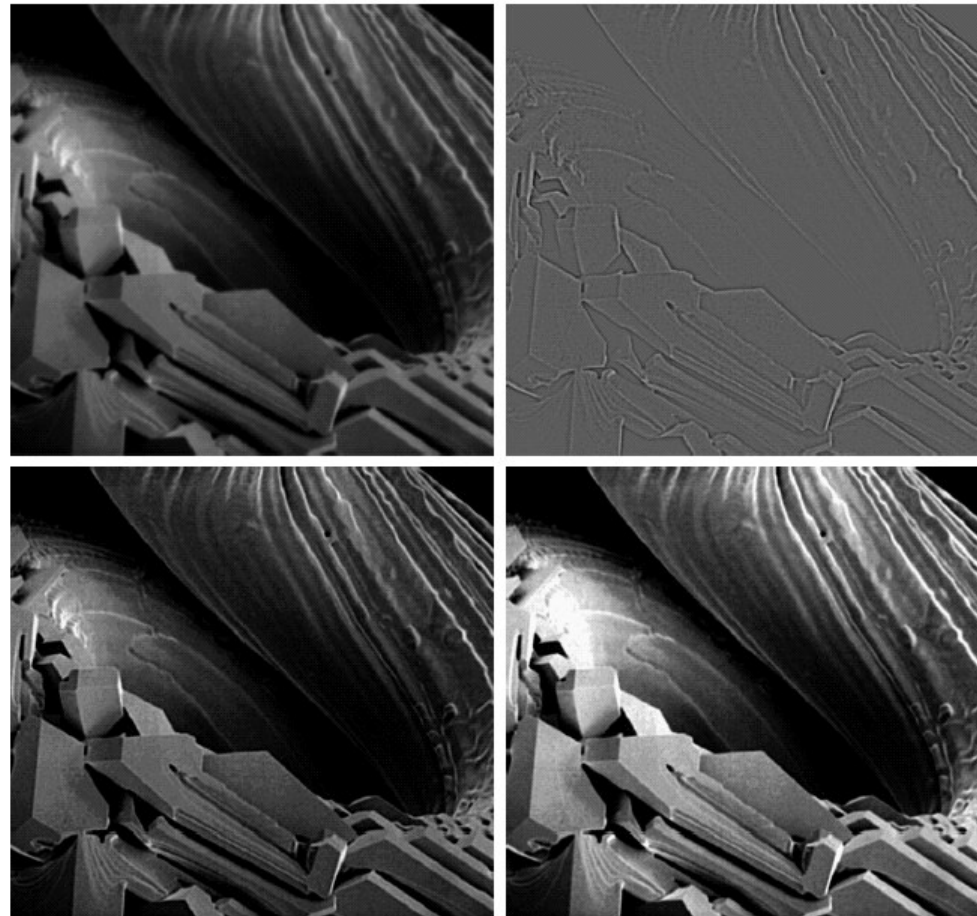


Image Enhancement in the Spatial Domain

Gradient Filter

Gradient

$$\nabla f = \left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \right]^T$$

$$\|\nabla f\| = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

Image Enhancement in the Spatial Domain

Gradient Filter

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$|\nabla f| \approx \left[(z_5 - z_8)^2 + (z_5 - z_6)^2 \right]^{1/2}$$

$$|\nabla f| \approx |z_5 - z_8| + |z_5 - z_6|$$

Robert's operator

1	0	0	1
0	-1	-1	0

$$|z_5 - z_9| \quad |z_6 - z_8|$$

prewitt

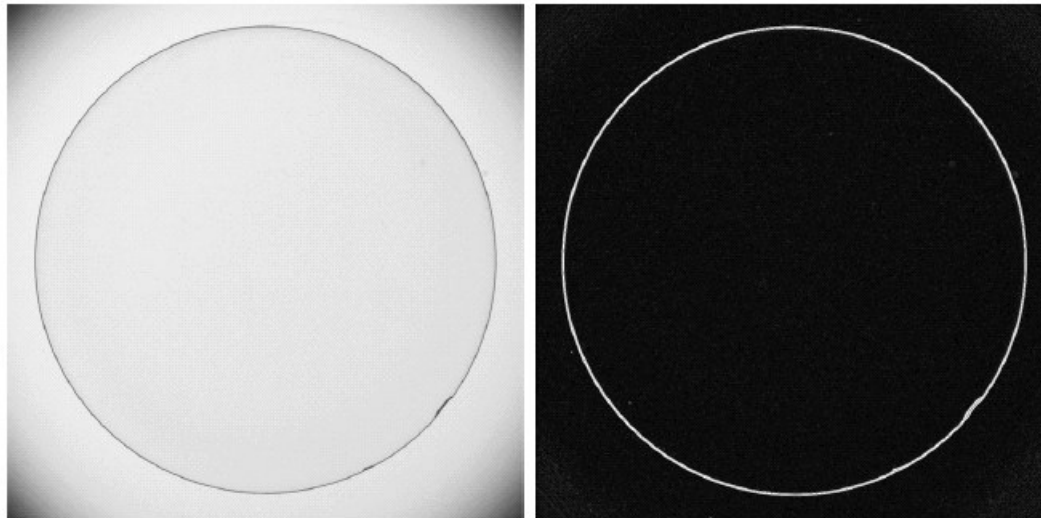
-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Sobel's

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Image Enhancement in the Spatial Domain

Gradient Filter: Example



a b

FIGURE 3.45

Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient.
(Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

Image Enhancement in the Spatial Domain

Laplacian Filter

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

	1	
1	-4	1
	1	

Image Enhancement in the Spatial Domain

Laplacian Filter: Example

a b
c d

FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)

